

# Application of Electric Circuit Analogies to Loudspeaker Design Problems\*

BART N. LOCANTHI

*California Institute of Technology, Pasadena, Calif.*

Electric circuit analogies are derived for three types of loudspeaker systems: direct radiator in an infinite baffle, direct radiator in a reflex enclosure, and horn loudspeaker. The data are in good agreement with data taken from experimental acoustical units.

**Editor's Note:** It is with pleasure that we publish this 1952 tutorial paper from the *IRE Transactions on Audio*. Mr. Locanthi has simply substituted the new I.S. units for the old c.g.s. system.

**INTRODUCTION:** Electrical engineers and physicists are frequently concerned with the task of obtaining solutions to electromechanical problems. These people are generally well versed in electric circuit theory. By transforming all of the mechanical "constants" of an electromechanical system to their equivalent electrical quantities, an electric circuit analogy is developed from which the qualitative performance may be quickly judged. Furthermore, quantitative data may be obtained by making appropriate measurements in the electric circuit.

It will be demonstrated in the paper that electric circuit analogy data are in good agreement with data taken from the following experimental acoustical units: 1) direct radiator in an infinite baffle, 2) direct radiator in a reflex enclosure, and 3) horn loudspeaker.

Especially in view of the good agreement between the electric circuit analog data and those obtained from an experimental horn loudspeaker, which was treated partly as a lumped parameter system and partly as a distributed

parameter system, there can be little doubt as to the power of this type of analysis.

Electric circuit analogies are derived in this paper for the three types of loudspeaker systems described. The analog computer at the California Institute of Technology was used to obtain data from the electric circuit analogies.

Electric circuit analogies for electromechanical systems have been known for at least 50 years [1]. It is the purpose of this paper to demonstrate the application of electric circuit analogies to certain loudspeaker design problems. In particular, the direct radiator in an infinite baffle, the direct radiator in a "reflex" type enclosure, and the horn loudspeaker will be discussed. Throughout the discussion, the mobility analogy will be used (i.e., voltage represents velocity and current represents force). The only major assumptions that will be made to facilitate the analysis are 1) the cone or diaphragm has no resonant modes and moves as a uniform piston, and 2) the resistance losses in the suspension of the moving system are negligible compared to the radiation losses.

## CASE I: DIRECT RADIATOR IN AN INFINITE BAFFLE

A diagram of a direct radiator in an infinite baffle is shown in Fig. 1. Let

\* Presented at the 1952 IRE National Convention, March 3-6, New York. Reprinted from *IRE Transactions on Audio*, vol. PGA-6, March 1952.

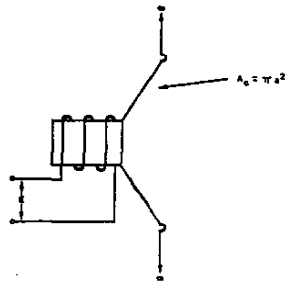


Fig. 1. Direct radiator loudspeaker in infinite baffle.

- $M$  = total effective mass of moving system (i.e., cone, voice coil, and effective air mass), kg
- $R_a$  = radiation resistance in mechanical ohms,  $N \cdot s/m$
- $C_m$  = effective compliance of suspension and back enclosure, if any,  $m/N$
- $i$  = applied current to driving coil, amperes
- $l$  = wire length of driving coil, meters
- $T$  = flux density in region of driving coil tesla
- $R$  = dc resistance of driving coil, ohms
- $W_a$  = acoustic power radiated, watts
- $x$  = displacement of moving system, meters
- $L$  = inductance of driving coil, henrys
- $E$  = applied voltage to driving coil, volts

In all the following analogies a voltage generator of zero source impedance will be assumed. However, it should be clear that an amplifier of finite source impedance may be represented by an amplifier of zero source impedance in series with the appropriate impedance. Insofar as the overall system performance is concerned, the amplifier source impedance may be considered as an additional element of the series electrical impedance ( $R_1 + j\omega L_1$ ).

The following two equations then describe the system [1], [2]:

$$L \frac{di}{dt} + R_1 = (E - E_1) \quad (1)$$

$$\frac{M}{(Tl)^2} \frac{dE_1}{dt} + \frac{R_a E_1}{(Tl)^2} + \frac{1}{C_m (Tl)^2} \int E_1 dt - i = 0. \quad (2)$$

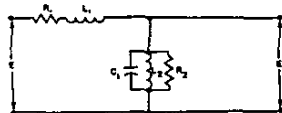


Fig. 2. Direct radiator loudspeaker in infinite baffle.

The electric circuit which satisfies (1) and (2) is shown in Fig. 2, where

$$R_1 = R \quad L_1 = L \quad C_1 = \frac{M}{(Tl)^2}$$

$$R_2 = \frac{(Tl)^2}{R_a} \quad L_2 = C_m (Tl)^2 \quad E_1 = (Tl) \frac{dx}{dt}$$

$$W_R = \frac{E_1^2}{R_2} = i^2 R_a \quad \text{watts.}$$

The parallel combination of  $C_1$ ,  $L_2$ , and  $R_2$  may be said to represent the inverse mechanical impedance of this moving system multiplied by  $(Tl)^2$ . As the factor  $(Tl)$  is increased, the impedance of the parallel combination of  $C_1$ ,  $L_2$ ,  $R_2$  increases as  $(Tl)^2$  relative to the series electrical impedance of  $R_1$  and  $L_1$ . Clearly, then, if  $(Tl)$  is increased far enough,  $E_1$  will equal  $E$  for a relatively wide range of frequencies. The bandwidth for which  $E_1 = E$  increases as  $(Tl)^2$ . If one refers to Fig. 3, which shows the radiation resistance per square meter of a piston, set in an infinite baffle, as a function of the ratio of piston diameter to wavelength of radiated sound, he will see that  $E_1 = E$  is not a desirable condition for all frequencies.  $E_1 = E$  implies a constant velocity condition and clearly, in this case, for  $D/\lambda$  less than 0.5, the acoustic output will drop at the rate of 6 dB/octave. It might be interesting to note that  $D/\lambda = 0.5$  for a 15-inch radiator at approximately 440 Hz.

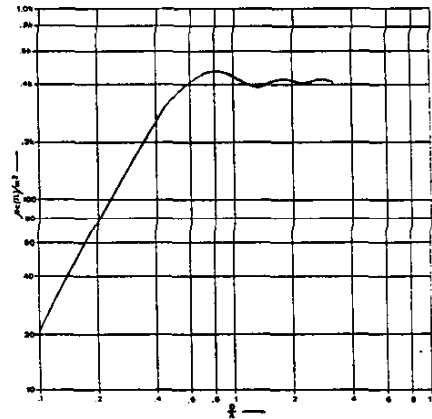


Fig. 3. Piston vibrating in infinite baffle.

Below  $D/\lambda = 0.5$ , the velocity of the radiator must increase inversely as the frequency if the acoustic power radiated is to be independent of frequency. The usual method for providing this frequency-velocity characteristic is to make use of the mechanical "resonance" of the loudspeaker. The insertion loss of this electro-mechanical equalizer is determined by the smallest  $D/\lambda$  down to which the response is to be uniform and the maximum variation in the low-frequency response which can be tolerated. If one sets the maximum variation in

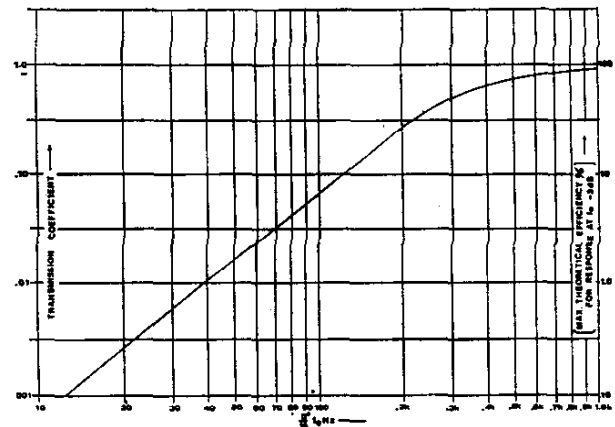


Fig. 4. Maximum transmission coefficient and theoretical efficiency, direct radiator loudspeaker.

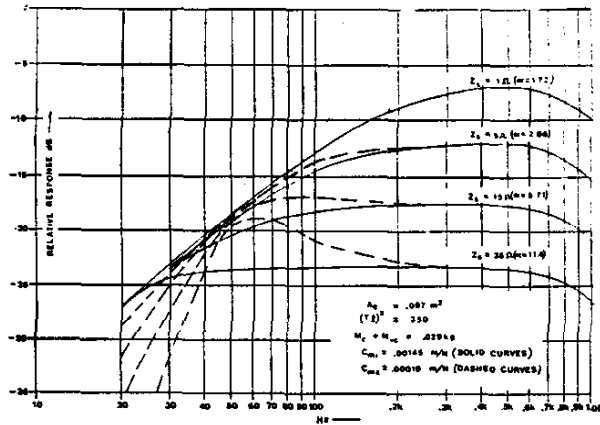


Fig. 5. Relative response 15" diameter, direct radiator loudspeakers.

low-frequency response at  $\pm 1.5$  dB, he may use the data in Fig. 3 and the concept of insertion loss to produce the curve shown in Fig. 4. The midrange (i.e.,  $D/\lambda = 0.5$ ) insertion loss is plotted as a function of  $D/15f_0$ , where  $D$  is the diameter of the piston radiator in inches and  $f_0$  is the low frequency at which the response is to be down 3 dB for infinite suspension compliance.

Fig. 5 shows the relative acoustic output as a function of frequency for several values of  $\alpha$ .

$$\alpha = \frac{R_1 \times 10^2}{(Tl)^2}$$

and for two different suspension compliances. The larger compliance was chosen to provide a system resonance in the infinite baffle of 20 Hz, while the smaller compliance was chosen to provide a system resonance of 55 Hz in the infinite baffle. The ordinate zero dB represents a midrange efficiency of 100 percent. It should be observed that only a slight improvement in response for the  $\alpha = 5.71$  curve was produced by using a mechanism having the smaller suspension compliance. For the  $\alpha = 10.44$  curve, the rise of 5 dB as system resonance would be held to be objectionable by many listeners.

Note the excellent correlation between the data shown in Figs. 4 and 5, even though two widely different system resonances were used. Fig. 4 is for some systems slightly pessimistic from an efficiency point of view.

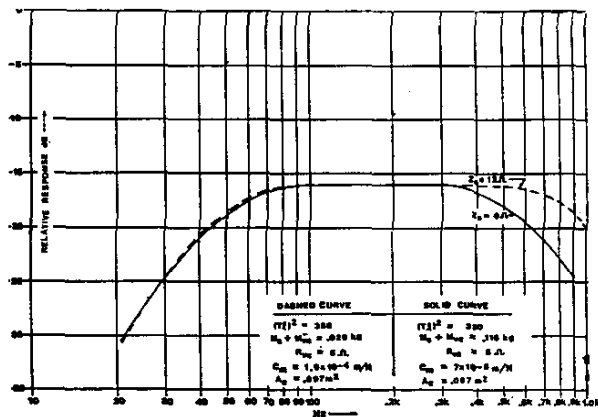


Fig. 6. Relative response of two different 15" direct radiator loudspeakers.

However, the extent to which this is so is small.

Fig. 6 shows the relative response curves obtained for two different 15-inch loudspeaker mechanisms possessing the same electromechanical coupling coefficients and the same system resonances (when mounted in an infinite baffle). The mechanisms differed in only one respect: the cone plus voice coil mass of one unit was four times heavier than that of the other. The heavy unit was driven by an amplifier which presented a zero source impedance while the lighter unit was driven by an amplifier which presented a 12-ohm source impedance.

## CASE II: DIRECT RADIATOR IN A REFLEX ENCLOSURE

A diagram of a direct radiator in a reflex enclosure is shown in Fig. 7. The port in a reflex enclosure may be treated as a zero length tube; the usual end conditions for both ends of a tube apply at the port. In most cases, the inner surface of the enclosure is covered with sound-absorbing material. At such low frequencies as are likely to be encountered near the Helmholtz resonance of the port, the absorption coefficient of the lining may be neglected. The major loss for the port is then radiation into the space away from the enclosure.

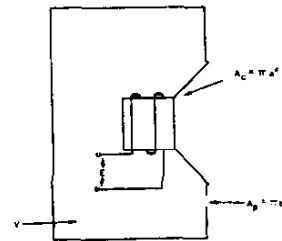


Fig. 7. Reflex enclosure loudspeaker.

The effective air mass acting at the port is approximately twice that due to the reactive component of the radiation impedance of one end of the tube, while the resistance offered by the port is the real part of the radiation impedance of the open end of the tube [3]. For constant pressure in the enclosure, the particle velocity at the port is proportional to the cone velocity and to the ratio of the cone area to the port area. The dimensions of the reflex enclosure are assumed to be small compared to the wavelength of sound for frequencies in the neighborhood of the Helmholtz resonance. The absorption coefficient of the material which lines the enclosure should be of sufficient magnitude at the resonant modes of the enclosure above the port resonance to damp them out.<sup>1</sup>

The port is coupled to the back of the cone by the compliant air of the enclosure. Let

- $\rho_0$  = density of air, kg/m<sup>3</sup>
- $\pi a^2 = A_c$  = cone area, m<sup>2</sup>
- $\pi b^2 = A_p$  = port area, m<sup>2</sup>
- $V$  = volume of enclosure, m<sup>3</sup>
- $c$  = velocity of sound, m/s.

<sup>1</sup> For a discussion which treats the variation of  $C_r$  from the uniform pressure case see [4].

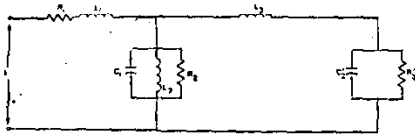


Fig. 8. Reflex enclosure loudspeaker without mutual impedance.

Then the compliance of the cone against the enclosure with the port closed satisfies the following equation:

$$C_v = \frac{V}{\rho_0 c^2 A_c^2}$$

Let  $M_{ap}$  represent the total air mass effective at the port, and  $R_{ap}$  the radiation resistance of the port. If one assumes that there is no coupling between the port and the cone outside the enclosure, he obtains the electric circuit analogy shown in Fig. 8 [5], where

$$\begin{aligned} R_1 &= R & L_1 &= L & R_3 &= \frac{(Tl)^2}{R_{ap}} \\ R_2 &= \frac{(Tl)^2}{R_a} & L_2 &= C_m (Tl)^2 & C_2 &= \frac{M_{ap}}{(Tl)^2} \\ R_3' &= \left(\frac{A_p}{A_c}\right)^2 R_3 & L_3 &= C_r (Tl)^2 & \frac{N_r}{N_p} &= \frac{A_c}{A_p} \\ C_1 &= \frac{M}{(Tl)^2} & C_2' &= \left(\frac{A_c}{A_p}\right)^2 C_2 \end{aligned}$$

It has been the author's experience that the impedance versus frequency curves for the analogy in Fig. 8 do not agree well with those of experimental units. The major differences appear to have been the frequencies at which the two low-frequency impedance maxima occur. The principal source of the discrepancy seems to be the omission of external coupling between the cone and port.

An approximate determination of the mutual impedance between the piston and the port may be determined in the following manner.

The pressure at a point  $p$ , distant  $R$  from the center of the vibrating piston and in the same plane (see Fig. 9), may be obtained by solving the following equation:

$$p(y) = \frac{i\rho_0\omega u_0 e^{i\omega t}}{2\pi} \int \int_s \frac{ds}{h} e^{-ikh}$$

The solution may be approximated by expanding  $e^{-ikh/h}$  in an infinite series and integrating the first four terms [6].

$$\begin{aligned} \frac{Zm'}{A_p} &= \rho_0 f \pi a^2 \left[ k - 1/6 k^3 \left( R_0^2 + \frac{a^2}{2} \right) \right] \\ &+ i\rho_0 f \left[ 4 \frac{a^2}{R_0} B \left( \frac{a^2}{R_0^2} \right) \right. \\ &- \frac{2k^2 a^2 R_0}{9} \left\{ \left( 5 + 3 \frac{a^2}{R_0^2} \right) K \left( \frac{a^2}{R_0^2} \right) \right. \\ &\quad \left. \left. - \left( 1 + 7 \frac{a^2}{R_0^2} \right) D \left( \frac{a^2}{R_0^2} \right) \right\} \right] \end{aligned}$$

where  $B$ ,  $K$ , and  $D$  are the complete elliptic integrals defined and tabulated in [7].

The inclusion of the mutual impedance terms modi-

fies the electric circuit analogy shown in Fig. 8 to the extent shown in Fig. 10, where

$$\begin{aligned} R_1 &= R & L_1 &= \bar{L} \\ L_2 &= C_m (Tl)^2 & C_1 &= \frac{M}{(Tl)^2} - C_3 \\ L_3 &= C_r (Tl)^2 & C_2' &= \frac{M_{ap}}{(Tl)^2} \left( \frac{A_c}{A_p} \right)^2 - C_3 \\ \frac{1}{R_2} + \frac{1}{R_4} &= \frac{R_r}{(Tl)^2} & C_3 &= \frac{2\rho_0 A c^2}{\pi^2 R_0 (Tl)^2} B \left( \frac{a^2}{R_0^2} \right) \\ R_3' + \frac{1}{R_4} &= \frac{R_{ap}}{(Tl)^2} \frac{A_c^2}{A_p} & R_4 &= \frac{(Tl)^2}{A_c^2 \rho_0 / k} \end{aligned}$$

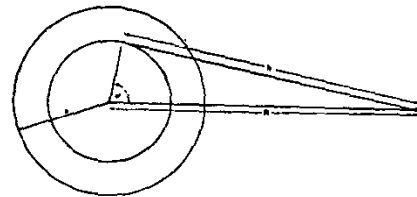


Fig. 9. Geometrical configuration.

It was found that this approximation modified the analogy shown in Fig. 8 so as to bring the impedance measurements of the electric circuit analogy into good agreement with those of experimental units. The inclusion of the mutual impedance into the electric circuit analogy for the reflex enclosure has been observed to produce the following differences from the electric circuit analogy which does not include the effect of the mutual impedance.

- 1) A reduction of the resonance frequency, which occurs above the port resonance, by 4 to 7 Hz out of an average of 65 Hz for several designs considered;
- 2) an increase of the resonance frequency, which occurs below the port resonance, by 2 to 4 Hz out of an average of 25 Hz for the several designs considered;
- 3) a sharper cutoff in response below the port resonance frequency.

No difference in the frequency at which the port resonance occurs has been observed between the electric circuit analogy which includes the effect of the mutual impedance and the electric circuit analogy which does not include the mutual impedance.

Figs. 11-14 show computed response curves for two different loudspeaker mechanisms mounted in two different reflex enclosures. Fig. 15 shows the computed relative acoustic output as a function of frequency for the same loudspeaker mechanism when mounted in two different types of enclosures. The amplifier source impedance was adjusted in each case to provide the same de-

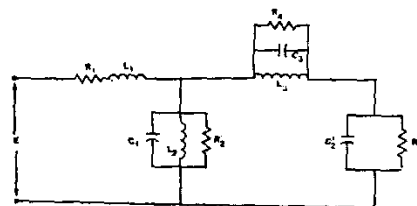


Fig. 10. Reflex enclosure loudspeaker with mutual impedance.

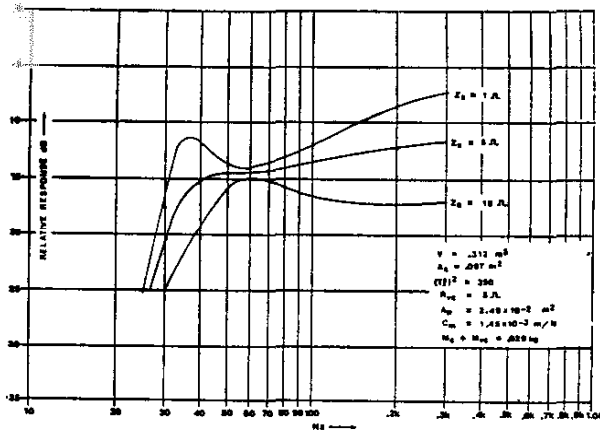


Fig. 11. Response curves (solid, Fig. 5) in reflex enclosure of 0.312 m<sup>3</sup> and port area 0.0249 m<sup>2</sup>.

gree of relative flatness in response. In the lower part of Fig. 15 are plotted the relative cone displacements as a function of frequency required to produce the response curves shown above. Note that a 6-dB improvement in overall efficiency is easy to attain with the reflex enclosure over the infinite baffle design and with no greater peak-to-peak displacement down to 40 Hz.

Many designs which provide a relatively high frequency port resonance (i.e., 70 to 100 Hz) produce considerable low-frequency distortion when driven at frequencies from 30 to 65 Hz. Below the port resonance, the acoustic loading of the cone or diaphragm is very low, large cone excursions ensue, producing very little radiation at the fundamental driving frequency and considerable harmonic distortion.

The electric circuit analogy is, of course, applicable to modified Helmholtz resonators in which a tube of finite length is used. The total mass effective at the port is then that due to both end corrections plus that due to air contained within the tube. The Helmholtz resonance frequency satisfies the following equation:

$$f_h \cong \frac{1}{2\pi} \sqrt{\frac{1}{M_{ep}C_r}} = \frac{bc}{2} \sqrt{\frac{3}{V(3\pi l + 16b)}}$$

where all parameters are as before and  $l$  is the axial port length.

In the case of enclosures of small volume, the area of the port required for a given low resonance frequency

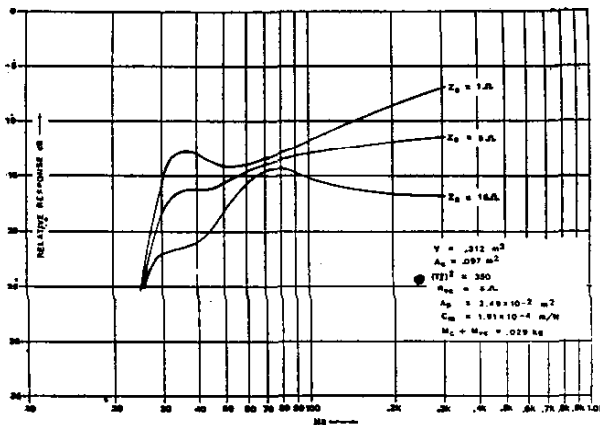


Fig. 13. Response curves (dashed, Fig. 5) in reflex enclosure of 0.312 m<sup>3</sup> and port area 0.0249 m<sup>2</sup>.

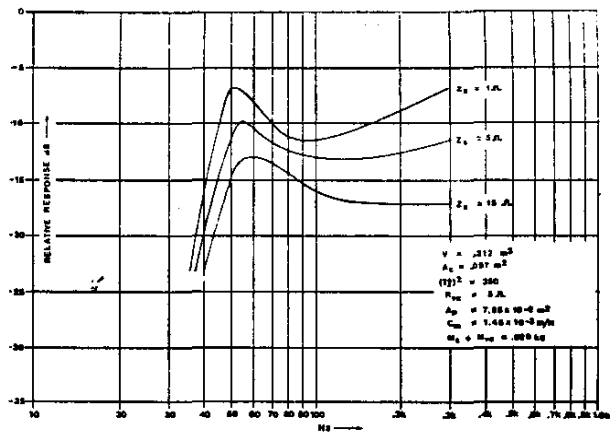


Fig. 12. Response curves (solid, Fig. 5) in reflex enclosure of 0.312 m<sup>3</sup> and port area 0.0755 m<sup>2</sup>.

may be so small that friction losses in the port may exceed radiation losses. Heavy, large diameter dummy cones with soft suspensions may be substituted for the air mass in the port to obtain low-frequency Helmholtz resonances in small enclosures.